1 Introduction

Procedures in the natural lumina of the human body are indispensable in minimally invasive diagnostic and therapeutic procedures [1]; however, complete controlled access to the entire gastrointestinal (GI) tract [2] and circulatory system [3] is challenging. A magnetically actuated soft inchworm robot concept recently presented in Ref. [4] can be exploited to produce controllable robotic therapeutic and diagnostic tools. As shown in Fig. 1, the robot consists of a deformable (soft) body between two magnets that are coaxially aligned with opposing polarity. A controllable rotating actuation magnet positioned outside of the patient, as illustrated in Fig. 1(a), induces a gait in the endoluminal robot that propels it through the lumen. The actuator magnet’s axis of rotation and position need to be kept approximately perpendicular to the lumen and roughly above the robot, respectively. The separation distance of the actuator and robot significantly affects the gait. The image sequence in Fig. 1(b) portrays the induced gait, which is similar to that of an inchworm caterpillar’s gait, shown for comparison in Fig. 1(c). First, there is an anchor-pull phase where the leading foot anchors and the lagging foot is pulled closer to the leading foot by contracting the body. This phase is followed by an anchor-push phase where the lagging foot is anchored and the leading foot is pushed forward by extending the body. Since the soft robot is mechanically simple, it can be adapted to various lumen sizes present in the body, and the external actuation magnet eliminates the need for complex mechanisms for locomotion within the robot. Although initial results show the potential for the robot to travel through the lumen, a thorough understanding of the physics and locomotion of the device is needed to develop and fully optimize the design. The focus of this paper is the modeling and analysis of the locomotion of the soft robot of Fig. 1. Dimensional analysis is then performed to efficiently study the effects of the various design variables on the robot’s motion. Endoluminal procedures in the GI tract traditionally involve pushing a long flexible scope from its proximal end [5, 6]. In the early 2000s, passive capsule-shaped cameras were first used to inspect the bowels and reach regions traditional scope methods could not, such as the small intestine [7]. Capsule endoscopy relies on peristalsis, so only uncontrolled antegrade travel is possible. Since the motion and orientation are uncontrolled, the camera may fail to capture images of regions of interest [8]. In order to tackle the aforementioned deficiencies, endoscopic and capsule devices are becoming more robotic with the addition of controllable actuation. Controllable capsule-like robotic devices will enable orientation and position control [9], including retrograde travel (i.e., against peristalsis). This will expand clinicians’ reach and capabilities during endoluminal procedures throughout the human body.

Many methods of endoluminal locomotion have been proposed, such as inchworm-like (anchor-pull and anchor-push) [10–13], legged [14–16], continuous track [17], worm screw [18–20], and vibration mechanisms [21]. The current challenge with endoluminal robots is creating a useful controllable device that can operate in a natural lumen for the duration of a procedure without damaging the lumen environment [22]. Battery-powered devices offer limited performance due to battery size, power requirements, and the time required for endoluminal procedures [23, 24]. Inductive power has been proposed and is feasible in the GI tract. With limited power available (310 mW with a cylindrical power receiver 12-mm long and 10-mm diameter [25]), however, it may not be feasible to power both the locomotion and on-board instruments. Due to

---

Keywords: medical robotics, robot design, soft robots
The soft endoluminal inchworm robot is driven by the magnetic interaction between the internal robot and an external actuator magnet located above the robot, as shown in Fig. 1. The robot moves due to alternating feet sticking and slipping as the robot’s body deforms and the feet rotate. This behavior is created by the nonuniform magnetic actuation field, which causes asymmetric magnetic forces and torques on each foot of the symmetric robot.

To develop the model, first, the magnetic interactions that drive the system are briefly described. Second, a static model of the soft robot that captures the interactions with the environment is developed. Next, an iterative process is created to determine the robot’s static equilibrium for a given actuator pose. Lastly, a discretized model is developed where the gait of the robot is determined by successive static equilibria and proper initial conditions.

2 Model

2.1 Magnetic Actuation Model. An external rotating magnetic field drives the soft robot in a lumen. For simplicity, it is assumed that all magnets in the model are perfect dipoles. This model is perfectly accurate for a uniformly magnetized sphere, and it is very accurate for an actual physical spherical magnet. For other shaped magnets (e.g., cubes, cylinders, rings), this model becomes increasingly accurate with greater distances [30]. The magnetic field $B$ at a relative position $r$ with respect to a dipole source $M$ is defined as

$$B(r) = \frac{\mu_0}{4\pi} \frac{3(M \cdot \hat{r})\hat{r} - M}{|r|^3}$$

where $\mu_0$ is the permeability of free space, $F$ is a unit vector in the direction of $r$, and all vectors are described with respect to a common coordinate frame as depicted in Fig. 1(a) [31].

The gait of the robot is driven by the interaction of the magnets in the robot with the actuation magnet. All of the magnets in the system impose forces and torques on each other. The force $F$ and torque $T$ on a dipole $M$ in a magnetic field $B$ are given by

$$F = (M \cdot \nabla)B$$

and

$$T = M \times B$$

These induced forces and torques cause the body of the robot to elastically deform. This deformation, along with sticking and slipping of the contact points on the robot’s feet, cause the robot to move in a deterministic direction within the lumen.

2.2 Locomotion Model. Using the pairwise dipole–dipole interactions between all magnets in the system, the net magnetic force and torque on each of the robot’s magnets can be calculated and used in a model describing the robot’s deformation and movement through the environment. An accurate model will enable exploration of design variable trends (e.g., body length, dipole moments) and simulation of the robot’s performance prior to physical realization.

By examining the physical robot in motion, it can be observed that the robot exhibits an inchworm-like gait (anchor-pull anchor-push) [29], as illustrated in Figs. 1(b) and 1(c). It has been found...
experimentally that the robot operates most predictably at low frequencies (typically 1–3 Hz) [4]. At this speed, during the entire gait cycle, at least one contact point between the environment and robot is not slipping. At these low frequencies, if the actuator magnet’s rotation is stopped, the robot holds its position with minimal inertial effects. Due to these characteristics, a quasistatic modeling approach is used to capture the motion of the gait using classical beam theory and mechanics.

To model the quasistatic behavior of the robot, the actuator magnet’s rotation is stepped through discretized angles (and positions if it is not fixed). At each time-step $k$, the actuator dipole angle $\theta$ is increased by an angle step $\phi$, such that $\theta[k+1] = \theta[k] + \phi$. At each $\theta[k]$, an iteration process is used to find the static equilibrium of the robot. The robot’s state of equilibrium is not unique and relies on boundary conditions based on the previous solution from time-step $k - 1$.

This iterative process involves finding an error term $\epsilon$ (described in Sec. 2.2.5), calculated for each iteration $i$. The $\epsilon$ term is energy-like and gives insight into the off-equilibrium robot energy. A proportional-integral-derivative (PID) controller is used to drive the error term to zero, and thus the robot to a state of equilibrium, by changing the vertical boundary conditions of three points of the robot: the bottom of the two feet ($\eta_A, \eta_B$) and the body of the robot ($\eta_C$).

### 2.2.1 Model Setup

The robot and environment are assumed to be composed of cylindrical elements that are symmetric about the plane swept by the rotating actuator dipole. This allows the model to be simplified to two dimensions in the $x/y$-plane as shown in Fig. 2(a). Since the soft robot is reminiscent of an inchworm, the cylindrical ends are referred to as “feet” and the midsection as a “body.” In the model, the feet are represented as rigid rectangles and the body as a deformable beam. The left foot is referred to as $A$ with mass $m_A$, and the right foot is referred to as $B$ with mass $m_B$. The body of the robot has mass $m_{body}$. Since the body’s length is much longer than its thickness, classical beam theory is used to approximate the body deformation. It is also assumed that the feet remain perpendicular to the ends of the body. The lumen, which the soft robot is traveling in, is represented as a floor and a ceiling. The floor is a straight horizontal line at a distance $F$ from the actuator magnet location. It is assumed that the robot’s weight alone does not significantly deform the lumen it is in. In the case where the magnetic forces or torques are causing the robot to make contact with both the floor and ceiling surfaces, the lumen is allowed to deform. In the model, the ceiling can deform, whereas in real life, it is expected that the ceiling and floor would deform at approximately half of what the model predicts for ceiling deformation. The main difference here being the vertical displacement of the robot, but not the gait behavior itself. The undeformed location of the ceiling $\mathcal{C}$ is one-lumen diameter $d_1$ above $F$. Each possible point of contact—foot $A$, foot $B$, and the body—with the ceiling is modeled as an independent linear spring with stiffness $k_A$. Each point of contact with the ceiling and floor is able to make and break contact as the robot deforms and moves.

A free body diagram of the soft robot is shown in Fig. 2(b). All force vectors are decomposed into their $x_c$ and $y_c$ components and torque is along the $z_c$ axis. To represent a single dimension of a point, the subscript ends in $x$, $y$, or $z$. The world frame of the model is represented by a subscript $o$. The lumen–environment axis is along the $x_o$ axis. Forces and torques on the feet act at the center of gravity of the foot ($A_{o}$ and $B_{o}$). There is a vertical force (along $y_o$), horizontal force (along $x_o$), and a torque (along $z_o$) acting at the center of gravity for each foot. The vertical forces, $F_{A_o}$ and $F_{B_o}$, combine the weight of the foot with the vertical magnetic forces. The horizontal forces, $F_{A_x}$ and $F_{B_x}$, are the horizontal components of the magnetic forces. The torques, $T_{A_y}$ and $T_{B_y}$, are the combined magnetic torques. The weight of the robot’s body section is represented as a constant distributed load $q$.

Each segment of the robot has two possible reaction forces: one with the ceiling and one with the floor. The ceiling contact points, $A_c$ and $B_c$, are taken as the highest corners of the foot, and the floor contact points, $A_f$ and $B_f$, are the lowest corners. Each foot contact location has a reaction force $R$ and friction $f$. For simplification, it is assumed that the reaction forces act at the center of gravity of the robot body, $C_{o}$. The body is also assumed frictionless since the friction at the feet dominates the inchworm gait.

The three ceiling reaction forces act independently. Without loss of generality, the reaction force between the ceiling and foot $A$ is

$$R_{A_o} = \begin{cases} 0 & \text{if } A_{o} \notin \mathcal{C} \\ \epsilon_A R_{k_c} & \text{otherwise} \end{cases}$$

where $\epsilon_A = A_{o} - \mathcal{C}$ is the displacement of the spring defined for foot $A$. The reaction forces with the floor are found using the equilibrium equations described in Sec. 2.2.2.

For each iteration at time-step $k$, it is assumed that one contact point on one foot is fixed in the $x_c$ direction from the prior time-step $k - 1$, and friction forces are computed for the non-fixed contact points. The friction force between the foot and the ceiling is

$$f_{A_c} = \delta_{A_c} \mu_c R_{A_c}$$

where $\mu_c$ is the static friction coefficient, and $\delta_{A_c} \in [-1, 1]$ encodes the direction and proportion of static friction used. For each iteration $i$ and for each of the non-fixed contact points, the $\delta$ term is updated based on if a contact point has moved in the $+x_c$ or $-x_c$ direction relative to the prior time-step $k - 1$. The update equation for $\delta$ with respect to $A_c$ is

$$\delta_{A_c, i+1} = \begin{cases} \delta_{A_c, i} + \Delta & \text{if } A_{o,k} < A_{o}[k - 1] \\ \delta_{A_c, i} - \Delta & \text{if } A_{o,k} > A_{o}[k - 1] \\ \delta_{A_c, i} & \text{otherwise} \end{cases}$$

Fig. 2 Two-dimensional representation of the soft robot and lumen environment. (a) Mechanical system model. (b) Free body diagram of the robot. (c) Free body diagram of foot $A$. (d) Free body diagram of the body, cut at an arbitrary location.
where $\Delta$ is a tuning gain that determines the amount $\delta$ can change in each iteration. The point $A_{g,i}$ is found at the end of each iteration $i$ when the robot placement in the environment is determined (Sec. 2.2.4). The point $A_{g,i}$ is the final value of $A_{g,i}$ at time-step $k$. To smooth the friction values and reduce jittering of the solution, an infinite impulse response filter is used on all non-fixed contact point friction forces. The friction force at the fixed contact point is found using the equilibrium equations shown next.

### 2.2.2 Equilibrium Equations

The equilibrium equations used to solve for the unknown forces at each iteration $i$ are described. First, during each iteration, the forces and torques from the dipole–dipole interactions are calculated based on the new positions of the magnets using the following equilibrium equations (refer to free body diagrams shown in Fig. 2(b)):

\begin{align}
\sum F_i &= f_{MV} + f_{BE} + f_{Fx} + f_{Bx} + F_{Ah} + F_{Bh} = 0 \quad (7) \\
\sum F_i &= -R_{MV} - R_{BE} - R_{C'M} - R_{Ah} - R_{Bh} \\
&+ R_{Bx} - R_{C'M} - F_{Av} - F_{Bv} - q_{Bh} = 0
\end{align}

\begin{align}
\sum M_{A_{x,i}} &= (A_{x,i} - A_{m})R_{A_{x,i}} + (A_{ay,i} - A_{fy,i})f_{A_{x,i}} \\
&+ (A_{ay,i} - A_{fy,i})R_{Wx} + (A_{ay,i} - A_{fy,i})R_{Wx} \\
&+ (B_{x,i} - A_{m})F_{Ax} + (A_{ay,i} - B_{x,i})F_{Bx} \\
&+ (A_{ay,i} - B_{x,i})R_{Wx} + (A_{fy,i} - B_{x,i})R_{Wx} \\
&+ (A_{ay,i} - B_{x,i})F_{Ax} + (A_{ay,i} - B_{x,i})F_{Bx} \\
&+ (C_{x,i} - A_{m})(R_{C'M} - R_{Bv}) \\
&+ (A_{ay,i} - B_{x,i})/2q_{Bh} + T_{Am} + T_{Bm} = 0
\end{align}

along with the displacement boundary conditions, the reaction forces $R_{A_{x,i}}$, $R_{Bx}$, and $R_{C'M}$, and the friction force of the fixed point can be found.

### 2.2.3 Robot Deformation

The body of the robot is assumed to be homogeneous and elastic. The deflection is found by solving

\begin{equation}
\frac{d^2}{dx^2}B_y = \frac{M_y(x)}{EI}
\end{equation}

where $M_y(x)$ is the moment along the body, $B_y$ is the vertical offset of each discretized point along the centerline of the body beam, $E$ is Young's modulus, and $I$ is the second moment of area of the body. To determine $M_y(x)$, a “cut” is made where the beam connects to the foot at point $A_{h,i}$ for foot $A$ (shown in Figs. 2(c) and 2(d)). The torque $T_{cut}$ and forces $V_{cut}$ and $N_{cut}$ on the ends of the body can be found by

\begin{align}
T_{cut} &= (A_{h,i} - A_{m,i})R_{Wx} + (B_{x,i} - A_{m,i})f_{A_{x,i}} \\
&+ (A_{x,i} - A_{m,i})R_{Wx} + (B_{x,i} - A_{m,i})f_{A_{x,i}} \\
&+ (B_{x,i} - A_{m,i})F_{Ax} + (B_{x,i} - A_{m,i})F_{Bx} + T_{Am}
\end{align}

\begin{equation}
V_{cut} = R_{Wx} - R_{A_{x,i}} + F_{Av}
\end{equation}

and

\begin{equation}
N_{cut} = -f_{A_{x,i}} - f_{Bx} - F_{Ah}
\end{equation}

The moment along the beam can now be found using (Fig. 2d)

\begin{equation}
M_y(x) = -1/2q_{Bh} - V_{cut}x_{B} - T_{cut}
\end{equation}

Plugging the above expression into Eq. (10), the following result is obtained:

\begin{equation}
B_y = (-1/24q_{Bh} - 1/6V_{cut}x_{B} - 1/2T_{cut}x_{B} + c_1x + c_2)/(EI)
\end{equation}

where the boundary conditions $\eta_1$ and $\eta_2$ can be used to solve for the constants of integration $c_1$ and $c_2$.

### 2.2.4 Robot Placement in Environment

Once the body deformation is found at the end of the $i$th iteration, the robot’s location in the environment can be determined. First, the feet are added perpendicular to the body. Then, the robot’s location in the world frame is found based on which reaction force is larger. The foot corner location with the highest reaction force is kept fixed in the $x_c$ displacement from the same corner in the previous time-step. The vertical offset of each foot is determined by $\eta_1$ and $\eta_2$. Due to the unknown amount of foot rotation between iteration $i$ and $i+1$, only $\eta_1$ and $\eta_2$ can be fixed in the solution at $i$th iteration. The $\eta_{C,i}$ term is only used as a boundary condition when determining body deflection.

### 2.2.5 Error Term

The error term, $\xi$, is an energy-like function that gives insight into the off-equilibrium energy of the robot. To find the equilibrium, the inputs to the model are the displacements $\eta_{A,i}$, $\eta_{B,i}$ and $\eta_{C,i}$. The $\eta_{A,i}$ and $\eta_{B,i}$ terms are used to define the vertical offsets of foot $A$ and $B$ during the $i$th iteration. The $\eta_{C,i}$ term is used as a boundary condition to solve for the over constrained beam deflection of the body. The error term $\xi$ and input term $\eta$ have different equations for the feet and body of the robot due to the different mechanics of the sections.

For feet $A$ and $B$, the inputs $\eta_{A,i+1}$ and $\eta_{B,i+1}$ are found using the following PID update law (shown for foot $A$ and is analogous for foot $B$):

\begin{equation}
\eta_{A,i+1} = k_p(\xi_{A,i}) + k_d(\xi_{A,i} - \xi_{A,i-1}) + k_i\sum_{j=0}^{i} \xi_{A,i-j}
\end{equation}

where $k_p$, $k_d$, and $k_i$ are constants, and $\eta_{A,i} \in [0, \infty)$. For the body, instead of an integral term, the $x_c$ position at the center of the beam is used ($C_{x,y,i}$), hence

\begin{equation}
\eta_{C,i+1} = k_p(\xi_{C,i}) + k_d(\xi_{C,i} - \xi_{C,i-1}) + C_{x,y,i}
\end{equation}

At equilibrium, the error term $\xi$ is zero. The error term is based on the assumption that for the robot to be in static equilibrium, the following statement is true for each section (foot $A$, foot $B$, and body) of the robot: (1) the section’s reaction force with the floor is zero or (2) the section’s reaction force with the floor is positive and the section is touching the floor. The equations work to drive the robot to satisfy the assumption by independently changing the height of each foot and the midpoint of the body based on the reaction forces solved for in Eqs. (7) through (9).

The error term for the two feet of the robot, $\xi_A$ and $\xi_B$, is found with the multiplication of a term related to distance and a term related to force, $\alpha$ and $\beta$, respectively. The assumption above has two conditions that are both represented in the error function by $\alpha$ and $\beta$. Condition (1) is based on direction and displacement, which is captured with the $\alpha$ term. The other is based on the reaction force, which is captured with $\beta$. The error equation (shown for $A$ and is analogous for $B$ and $C$) is given by

\begin{equation}
\xi_{A,j} = \alpha_{A,i}\beta_{A,j}
\end{equation}

The $\beta$ term is related to the magnitude of the floor reaction force (shown for $A$ and in analogous for $B$ and $C$):

\begin{equation}
\beta_{A,j} = 1 - \frac{F_n - \left(\text{sat}(R_{A_{x,i}} \pm F_n)\right)}{F_n}
\end{equation}

where $F_n$ is a parameter to normalize $\beta$. This term needs to be tuned depending on the magnitude of the forces in the model. The $\beta \in [0, 1]$ term dictates how much influence the force has on
the $\xi$ term. If the force is large, then $\beta = 1$ and if the force is zero, then $\beta = 0$.

The $\alpha$ term relates to condition (2) and determines the direction the distance $\eta$ needs to change such that the boundary conditions for the ceiling and floor are met: reaction forces should be positive and if there is a positive reaction force, there needs to be contact there. The $\alpha$ term for the feet is (shown with foot $A$ and is analogous for foot $B$) given by

$$
\alpha_{A,j} = \begin{cases} 
(\xi - A_{y,f})k_1 & \text{if } R_{A,F} > 0 \\
(\xi - A_{y,f})k_1 & \text{if } R_{A,F} < 0 \text{ and } A_{y,f} < \xi \\
-k_{y, A}k_2 & \text{if } R_{A,F} < 0 \text{ and } A_{y,f} > \xi \\
0 & \text{otherwise}
\end{cases} 
$$

(20)

where the $k_1$ and $k_2$ terms are used to scale the influence on $\xi$. In the above equation, there are three cases where $\alpha$ is nonzero, explained below with respect to foot $A$, but foot $B$ is found in an analogous manner:

1. If the reaction force with the floor $R_{A,F}$ is positive, then $\alpha_A$ is based on the distance from the bottom of the foot $A_{y,f}$ to the floor $\xi$.
2. If the reaction force with the floor $R_{A,F}$ is negative and the foot is not touching the ceiling, then $\alpha_A$ is based on the distance from the top of the foot $A_{y,c}$ to the nominal ceiling $\xi$.
3. If the reaction force with the floor $R_{A,F}$ is negative and the foot is already touching the ceiling, then $\alpha_A$ is based on how much the spring displacement $\epsilon_A$ would need to increase to balance out the reaction force with the floor.

Next, the error term for the body is explained. The $\alpha$ term for the midsection is defined differently than the feet. This is due to the system being very responsive to changes in this boundary condition.

It was found that the following definition worked well:

$$
\alpha_{C,j} = \begin{cases} 
k_3 & \text{if } R_{C,F} > 0 \text{ and } \min(B_{y}) - \frac{db}{2} < \xi \\
-k_3 & \text{if } R_{C,F} > 0 \text{ and } \min(B_{y}) - \frac{db}{2} > \xi \\
-k_{y,c} - R_{C,F}k_4 & \text{if } R_{C,F} < 0 \text{ and } \max(B_{y}) + \frac{db}{2} > \xi + \epsilon_c \\
k_3 & \text{if } R_{C,F} < 0 \\
0 & \text{otherwise}
\end{cases} 
$$

(21)

where $\min(B_{y})$ and $\max(B_{y})$ are the lowest and highest points on the centerline of the body $B_y$, respectively, $k_c$ is the spring constant of the ceiling, and $k_3$ and $k_4$ are gains that can be adjusted. It is noted that in many scenarios, the robot body does not contact the environment during travel. If this is the case, for simplicity the $\eta_i$ and $R_{C,F}$ terms could be neglected in the model.

2.2.6 Convergence. The output of the model converges when the deformation of the beam is sufficiently low between iterations, that is

$$
\mathcal{E} = \text{norm}(B_{y,i+1} - B_{y,i}) \leq \mathcal{E}_{\text{conv}}
$$

(22)

Once $\mathcal{E}$ is below threshold $\mathcal{E}_{\text{conv}}$, the static model has converged and the time increments to $k + 1$. In some cases, the model oscillates between two different equilibrium points and a maximum number of iterations $i_{\text{max}}$ is set for this case.

3 Model Validation

An experimental setup was created to compare the model predictions to physical experimental results. The experimental setup is
shown in Fig. 3(a). To validate the model, two environments are tested. First, the model output is compared to the robot in a rigid 6-mm diameter polycarbonate tube. Second, the model is compared to the robot actuated on an *ex vivo* pig intestine. The actuator magnet has a dipole strength, $M_a$, of 205 A m$^2$ (K&J Magnetics, D20X8-N52). The height of the actuator magnet above the robot can be adjusted. In each trial, the robot starts directly underneath the actuator magnet at $x_0 = 0$, then the actuator magnet was rotated clockwise, causing the robot to propel itself in the +x direction. An infrared (IR) motion-capture system was used to track the position of the robot. Two IR markers, one on each foot, were added to the robot to enable tracking, as shown in Fig. 3(b). The soft robots used in both experimental setups were fabricated with a vacuum injection silicone mold. The modulus of elasticity was experimentally determined by securing the robot as a horizontal cantilever beam and measuring the deflection when a small weight was added. The modulus of elasticity of the body was found to be 1370 kPa. The magnets in the feet each have a dipole moment of 0.011 A m$^2$ (1.17 × 10$^5$ A/m, K&J R211-N52).

In the polycarbonate lumen, the model predictions are compared to experimental results for four different configurations. The soft robot used has feet with a diameter of 5 mm and a length of 2.2 mm. The length of the body section is 20 mm, and the body diameter is 2.5 mm. The tube is filled with water to better represent a biological lumen. The static friction coefficient between the silicone robot and lumen is experimentally determined to be 0.75. To approximate the interaction between the silicone feet and effectively rigid lumen environment, the environment spring stiffness was set to 80 N/m. The rest of the design parameters are aligned with the nominal design described in Table 1. The experiments are conducted at four different separation distances between the actuator magnet’s axis of rotation and floor of the lumen, $F$: 23 cm, 25 cm, 27 cm, and 29 cm. At each $F$, 30 trials are performed where the robot is driven from $x = 0$ to 100 mm. The results are shown in Fig. 4. The raw position data from the IR camera motion-capture system are downsampled to 1 Hz (one measurement per gait cycle) and the velocity of the robot is calculated with central differentiation. The velocity data of the 30 trials are binned for each 1 mm of travel along the $x_0$ axis. The solid black line is the average velocity of the physical robot over 30 trials. The gray shaded areas are ±1 standard deviation around the average velocity of the physical robot. To determine the model simulation velocity, the position data are downsampled to 1 Hz, and the robot’s velocity is calculated with central difference differentiation. The dashed line is the model results for the velocity of the robot. Overall, the developed model captures the changing velocity profile of the gait of the soft robot as it travels away from the actuator magnet at four different experimental setups in this rigid lumen.

Next, model predictions are compared to an *ex vivo* pig intestine environment, adding additional compliance under the feet of the robot. The test environment is shown in Fig. 5(a). The pig’s small intestine is cut along the length and laid over a half-cylindrical trough (to enable visualization/localization), which is submerged in water. The friction coefficient is experimentally determined to be 2.7. The soft robot used has feet with a diameter of 5 mm and a length of 2 mm. The length of the body section is 20 mm, and the body diameter is 2 mm. The separation distance between the actuator magnet’s axis of rotation and floor, $F$, is 200 mm. The rest of the properties are the same as in the prior experiment. The robot starts directly underneath the actuator magnet ($x = 0$) and is driven in the +x direction 80 mm (Fig. 5(b)). The test was run 60 times, and the results are shown in Fig. 5(c); the model prediction is shown with a dashed line. The lumen diameter and ceiling properties are not used for this model because the modeled environment is a trough.

### 4 Impact on Design

The experimentally validated model is now used to explore the influence of the various design parameters on the robot’s performance. Prior work has not explored design trends and performance. For example, in Ref. [4], the forces and torques on a robot during the gait are shown in detail; however, the effects of parameters on performance is not known. A target environment will likely impose constraints on specific parameters, such as actuator magnet minimum separation from the robot or material of the soft robot. To support optimization of the robot for an environment, an understanding of the design trends is crucial.

#### 4.1 Dimensional Analysis

The step size $S$ per revolution of the actuator magnet is chosen as the dependent output variable.

---

**Table 1** The chosen repeating variables, nonrepeating variables, and dimensionless $\Pi$ groups of the modeled system

<table>
<thead>
<tr>
<th>Repeating variables</th>
<th>Symbol</th>
<th>Units</th>
<th>Nominal design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Env. lumen diameter</td>
<td>$d_e$</td>
<td>m</td>
<td>6.0e-3</td>
</tr>
<tr>
<td>Env. ceiling stiffness</td>
<td>$k_e$</td>
<td>kg/m$^2$</td>
<td>8.0e1</td>
</tr>
<tr>
<td>Acceleration of gravity</td>
<td>$g$</td>
<td>m/s$^2$</td>
<td>9.81</td>
</tr>
<tr>
<td>Env. magnetic permeability</td>
<td>$\mu_s$</td>
<td>kg m/s$^2$ A$^2$</td>
<td>4.0 × 10$^7$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nonrepeating variables</th>
<th>Symbol</th>
<th>Units</th>
<th>Nominal design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step size</td>
<td>$S$</td>
<td>m</td>
<td>$\Pi_0 = S/d_e$</td>
</tr>
<tr>
<td>Actuator dipole moment</td>
<td>$M_a$</td>
<td>A m$^2$</td>
<td>$\Pi_1 = M_a^2 / (d_e^3 k_e)$</td>
</tr>
<tr>
<td>Robot body length</td>
<td>$l_b$</td>
<td>m</td>
<td>$\Pi_4 = l_b/d_e$</td>
</tr>
<tr>
<td>Robot foot length</td>
<td>$l_f$</td>
<td>m</td>
<td>$\Pi_5 = l_f/d_e$</td>
</tr>
<tr>
<td>Robot body diameter</td>
<td>$d_b$</td>
<td>m</td>
<td>$\Pi_6 = d_b/d_e$</td>
</tr>
<tr>
<td>Robot central-lumen diameter</td>
<td>$d_l$</td>
<td>m</td>
<td>$\Pi_7 = d_l/d_e$</td>
</tr>
<tr>
<td>Robot silicone density</td>
<td>$\rho_s$</td>
<td>kg/m$^3$</td>
<td>$\Pi_8 = \rho_s d_e^2 / k_e$</td>
</tr>
<tr>
<td>Robot modulus of elasticity</td>
<td>$E$</td>
<td>kg/(m s$^2$)</td>
<td>$\Pi_9 = E d_e^3 / k_e$</td>
</tr>
<tr>
<td>Robot magnetization</td>
<td>$\Psi_m$</td>
<td>A/m</td>
<td>$\Pi_{10} = \Psi_m d_e^2 / k_e$</td>
</tr>
<tr>
<td>Robot magnet/foot vol. frac.</td>
<td>$V_m$</td>
<td></td>
<td>$\Pi_{11} = V_m d_e^2 / k_e$</td>
</tr>
<tr>
<td>Robot dipole angle, foot $A$</td>
<td>$\theta_{A}$</td>
<td></td>
<td>$\Pi_{12} = \theta_{A}$</td>
</tr>
<tr>
<td>Robot dipole angle, foot $B$</td>
<td>$\theta_{B}$</td>
<td></td>
<td>$\Pi_{13} = \theta_{B}$</td>
</tr>
</tbody>
</table>

Note: The last column shows the parameter values of the nominal design used throughout for comparison of design trends. In the following figures, the nominal design is marked with a black star.
The velocity \( v \) is related to step size \( S \) by \( v = S \omega \), where \( \omega \) is the angular velocity of the actuator magnet in hertz. There are three main types of independent variables: environment variables, actuator magnet variables, and soft robot variables (Fig. 6). The environment variables include the diameter of the lumen \( d_e \), the lumen ceiling stiffness \( k_e \), the acceleration of gravity \( g \), the magnetic permeability of the environment \( \mu_e \) (which in practice will typically be the permeability of free space \( \mu_0 \)), and the static friction of the robot with the environment \( \eta_s \). Of these environmental variables, the system designer can only influence the static friction between the soft robot and the lumen environment (through a choice of material and surface property). The actuator magnet variables are fairly limited, comprising the distance from the dipole axis of rotation to the floor \( F \) and the strength of the actuator dipole moment \( M_a \) (note that, since the quasistatic step size of the soft robot is modeled, the angular velocity of the actuator magnet is not included as a relevant independent variable). Finally, the variables relating to the soft robot are numerous, including body length \( l_b \), foot length \( l_f \), body diameter \( d_b \), central-lumen diameter \( d_l \), foot diameter \( d_f \), soft-material density \( \rho_s \), foot magnet volume fraction (the fraction of the foot volume that is composed of the embedded magnet) \( V_m \), density of magnet \( \rho_m \), body modulus of elasticity \( E \), magnetic field strength \( \Psi_m \), dipole angle \( \theta_{MA} \) for foot \( A \), and dipole angle \( \theta_{MB} \) for foot \( B \).

With a total of 19 variables that describe the system (18 independent variables plus the output variable), comprising four primary dimensions (kg, m, s, and A), the Buckingham \( \Pi \) theorem [32] states that the physics of the system can be fully described by \( 19 - 4 = 15 \) dimensionless \( \Pi \) groups (i.e., 14 independent dimensionless variables plus the resulting dimensionless output variable). Four linearly independent variables each containing at least one of the four primary dimensions are chosen to be used throughout the nondimensionalization process. This set of four variables, called repeating variables, are used to create the nondimensional \( \Pi \) groups. For the system described, four environmental variables
outside of the system designer’s control create a suitable set of repeating variables: $d_i$ for $m$, $g$ for $s$, $k_e$ for $kg$, and $\mu_s$ for $A$. After this choice, the resulting $\Pi$ groups are constructed deterministically and are shown in Table 1. This proposed nondimensionalization, although not unique, seems to be a good choice for guiding future system designs for a given application area (i.e., set of lumen properties).

4.2 Justifying the Original Concept. In all of the prior work relating to the soft robot concept which is the focus of this work [4,29], only coaxial parallel and coaxial antiparallel arrangements of the magnetic dipoles embedded in the robot’s feet have been investigated. It was experimentally determined in Ref. [4] that coaxial antiparallel dipole moments produce more robot motion along the lumen than coaxial parallel dipole moments. However, it is not known if there are potential benefits to magnetizing the robot’s magnets in a different way, so here the coaxial antiparallel arrangement is briefly explored to see if it is truly optimal. Using the naive scaling method, the output is not predictable or decreasing $\Omega$. This is due to the timing of the robot can contract in the anchor-pull phase of the gait. Also, if the inter-magnetic forces between the actuator magnet and robot are too great, the robot will be pinned against the lumen wall, unable to move forward. The effects of $M_s$ and $V_m$ can be seen in $\Pi_3$ and $\Pi_{12}$ in Figs. 8(b) and 8(f).

4.4 Scaling. There are two intuitive ways to scale the robot to different lumen environments. The simplest way, which is referred to as the naive method, is to scale the robot’s length dimensions linearly with the target lumen environment diameter while keeping all other properties constant. This method would be simple to achieve and would not require changing the material properties of the robot. The second method is to use the $\Pi$ groups from Table 1 to scale the robot design nondimensionally by keeping $\Pi_1$ through $\Pi_{15}$ constant. Both methods are investigated using the model. The robot’s nominal design, with an environment lumen diameter $d_e$ of 6 mm, is scaled up to 12 mm (2x) and down to 0.06 mm (0.01x). Forty logarithmically spaced scales are modeled, and the performance for the two methods is compared in Fig. 9.

Using the naive scaling method, the output is not predictable without a model and will not necessarily follow similar trends if a different initial design is used. Scaling $d_e$ up to two times the nominal value of 6 mm, the robot output $\Pi_6$ decreases and the velocity increases. A decrease in $\Pi_6$ as lumen diameter increases means the velocity increased at a slower rate than lumen diameter. As the nominal $d_e$ is scaled down to about 0.6 mm (0.1x), $\Pi_0$ decreases 85%. From a scale factor of 0.1–0.03, there is a slight increase and decrease in $\Pi_0$. This is due to the timing of the independent variables that affect robot motion. Starting from the nominal design and varying only one independent parameter at a time, trends in (nondimensional) step size as a function of important $\Pi$ groups are shown in Fig. 8. A few trends that are significant in optimizing robot performance can be observed. The robot’s step size is primarily based on how much the robot can contract and expand the body along with the rotation of the feet. The effects of all the design variables on the robot motion can be roughly summarized into four categories:

1. Variables that decrease the flexural stiffness of the robot’s body lead to an increase in step size. Variables that affect flexural stiffness include: $d_i$, $d_s$, $E$, and $l_s$. The step size increases until the robot is too flexible to keep the feet from sticking together because of the magnets in the feet. Increasing $l_s$ increases the step size up to a point at which the length no longer adds much additional rotation of the feet due to the body contacting the lumen. The effects of $l_s$ and $E$ can be seen in $\Pi_4$ and $\Pi_{10}$ in Figs. 8(c) and 8(e).

2. Variables that increase the magnetic forces and torques on a given robot cause an increase in step size. This can be done by increasing $M_s$, $V_m$, and $\Psi_m$ (increasing $\Psi_m$ is usually not possible as the highest grade of magnets is often already used) or decreasing $\mathcal{F}$. This trend is limited by the amount of the robot can contract in the anchor-pull phase of the gait. Also, if the inter-magnetic forces between the actuator magnet and robot are too great, the robot will be pinned against the lumen wall, unable to move forward. The effects of $M_s$ and $V_m$ can be seen in $\Pi_3$ and $\Pi_{12}$ in Figs. 8(b) and 8(f).

3. Increasing forces that hinder the robot motion, such as $\rho_s$, $\rho_m$, and $s_0$, decrease the velocity. If the friction between the robot and lumen is too low, the robot will not be able to move. The effects of $s_0$ and $\rho_m$ can be seen in $\Pi_1$ and $\Pi_6$ in Figs. 8(a) and 8(d).

4. Increasing $d_i$ increases the step size the robot takes for a given foot rotation. Larger foot diameters lead to a more significant step with the same angle of rotation. This is, of course, limited by the environmental lumen diameter.

Of course, all these trends have limits and cannot be infinitely exploited. For example, if the modulus of elasticity of the body is continuously decreased (with the intent of causing the body to have greater deformation leading to a greater rotation of the feet and a larger step), at some point the body is not rigid enough to keep the two embedded magnets separated, and the feet of the robot will stick together.
inchworm gait changing through this region. At a scale factor of less than 0.03, $\Pi_0$ is near zero. With the naive method, the robot performance is not constant or intuitive throughout the change in size.

Using the nondimensional scaling method, the robot output $\Pi_0$ remains constant and predictable throughout the scaling, and the gait remains the same. The step size of the robot changes linearly with the lumen diameter. Using this method, the design variables need to adhere to the following trends to keep $\Pi_1$ through $\Pi_{15}$ constant: length variables change linearly with the lumen diameter, density changes with $d^{-2}$, modulus of elasticity changes with $d^{-1}$, magnetization changes with $d^{-0.5}$, and lastly the dipole moment changes with $d^{-0.5}$. The nondimensional scaling method results emphasize that the naive method is not a great way to drastically change the size of the robot; however, the nondimensional scaling trends would likely not be possible to strictly follow in practice. Nondimensional scaling trends would need to be used along with the design trends in Sec. 4.3 to adapt the robot to the constraints of a desired environment and application.

5 Discussion

The model developed in this paper uses a quasistatic approach to enable simulation of the robot and exploration of design parameters. An experimental setup was fabricated to validate the model. Two different environments were tested. In the polycarbonate tube, the robot’s experimental velocity profile for four setup configurations is compared to the model simulation results. For all four configurations tested, the model simulation results of the velocity profiles are within one standard deviation of the mean experimental velocity during travel. On the ex vivo pig intestine, the model predictions follow the experimental results closely but are not as accurate as the rigid lumen. This is likely due to the unmodeled characteristic of the environment, such as biological irregularities and velocity-dependent friction. From these results, it is concluded that the model can be used to predict the soft robot’s overall behavior and can be used to examine performance and design trends.

The first variable investigated is the alignment of the dipole moments in each foot of the robot. It was found that the best configuration of the dipole moments in the feet of the robot is a coaxial antiparallel configuration; whether the dipole moments both point inward or both point outward made no significant difference. The rest of the design variables’ trends were explored. It was found that the speed of the robot generally increases when changing parameters that: (1) increase robot deformation under fixed forces and torques, (2) increase the magnitude of the magnetic forces and torques imposed on the robot, (3) decrease the resistance to motion, or (4) increase the step size for a given foot rotation.

Fig. 8 Dimensionless trends for six II groups of the modeled soft robot. Each plot shows the value of $\Pi_e = S/d_e$ as a single independent II group is changed relative to a nominal design (indicated with a black star).
for a specific lumen environment, the design trends and nondimensional scaling trends will give insight into how to achieve the desired output in a chosen environment. If scaling a particular robot to a specific environment and a nondimensional scaling trend cannot be satisfied, another parameter may be adjusted to offset the effects (using guidance from the design trends). For example, if the robot needed to be scaled to the desired lumen but the material of the robot, and therefore the modulus of elasticity, could not change, it may be possible to achieve the desired stiffness in the body of the robot by altering the body diameter. This change would, of course, have other effects on the gait but may bring the robot closer to the desired performance.

6 Conclusion
This paper presented a detailed model and analysis of a magnetically actuated inchworm-like soft robot that can travel through a lumen environment. The model will help mature the recently proposed robot concept and enable a deeper understanding of crucial design variables and how they influence the gait. The model was experimentally validated with a comparison to physical experiments using a plastic lumen environment and an ex vivo pig intestine. The model was shown to capture the robot’s behavior well as it travels through the environments. To explore and analyze the design trends and scaling, nondimensional terms were derived using the Buckingham II theorem. Using these terms, extensive simulations were performed that enable an in-depth understanding of which variables were sensitive to change, and their effect on the gait. The main trend being the greater the deformation of the body, the faster the robot. The robot concept can potentially be utilized in various endoluminal procedures in the human body. In the future, the locomotion concept explored here may extend the reach of endoluminal procedures and aid in developing controllable endoluminal devices. Additionally, motivated by preliminary results for the integration of electroactive polymer actuation [29], an in-depth investigation will be performed to study further performance enhancements.

Acknowledgment
This work was supported by the National Science Foundation (Grant No. 1830958). The authors would like to thank Professor Xiang He at Beihang University for his advice and discussion about the soft robot model.

Conflict of Interest
There are no conflicts of interest.

Data Availability Statement
Data provided by a third party are listed in Acknowledgment.

References


