Minimum-parameter Adaptive Propulsion Matrix of Screw-type Magnetic Capsule Endoscopes

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INTRODUCTION

Capsule endoscopes are employed for non-invasive video-based inspection of the intestines. Active control of such capsules continues to be an active area of research [1]. The most popular and seemingly most viable method to actively traverse the length of the small intestine involves equipping the capsule with an internal permanent magnet and an external screw-like thread (which would be coated in something like ice for swallowing) and rotating the capsule with an externally applied magnetic field such that rotation is converted to translation via the screw threads [2]–[7]. Empirical models of this method of locomotion have been proposed [3], [5], as have analytical models [2], [4], [7]. The model commonly used to describe the locomotion of screw-type capsules in a lumen (see, e.g., [4], [7]) is

\[
\begin{bmatrix}
    v \\
    \omega
\end{bmatrix} =
\begin{bmatrix}
    \alpha & \beta \\
    \beta & \gamma
\end{bmatrix}
\begin{bmatrix}
    f \\
    \tau
\end{bmatrix} = P^{-1} \begin{bmatrix}
    f \\
    \tau
\end{bmatrix}
\]

(1)

where \( f \) (units N) and \( \tau \) (units N\cdotm) are the non-lumen force and torque applied to the capsule along the lumen (e.g., due to magnetism and gravity), \( v \) (units m\cdots\(^{-1}\)) and \( \omega \) (units rad\cdots\(^{-1}\)) are the resulting linear and angular velocity of the capsule through the lumen (see Fig. 1), and \( \alpha \), \( \beta \), and \( \gamma \) (with consistent units) are the parameters that make up the inverse of the symmetric propulsion matrix \( P \) [8]; it can be shown that both \( P \) and \( P^{-1} \) must be positive-definite. Advanced control systems can make use of the model of (1) for optimal propulsion and localization (e.g., as an a priori process model in a Kalman filter [7]). However, the locomotion properties of these screw-type capsules change substantially as a function of the properties of their lumen environment (e.g., lubrication, lumen diameter). We have come to realize that adaptive controllers capable of updating the propulsion matrix to better reflect the capsule’s current environment can lead to better controllers, but adaptive controllers come with their own challenges related to stability and convergence.

The model of (1) has the same structure used for flagella-like magnetic microswimmers [8]. Fluid viscosity appears linearly in each of the propulsion-matrix parameters such that it can be factored out, leaving purely geometry-dependent terms [8]. As such, the propulsion matrix describing a given microswimmer has only a single parameter that can vary as a function of the fluid environment, since the geometry-dependent terms are constant. Given the other similarities between the models used for magnetic microswimmers and screw-type capsules, in this paper we experimentally investigate if the propulsion matrix (and its inverse) of a screw-type capsule can be expressed as the product of a constant positive-definite symmetric matrix and a single scalar parameter that varies as a function of the environment.

MATERIALS AND METHODS

Experiments were conducted to determine the propulsion matrix for a screw-type capsule (mass 10.1 g, length 42.0 mm, diameter 13.5 mm without thread, 1 mm thread height, 13.5 mm thread pitch, rigidly embedded 4.76 mm cubic NdFeB Grade-52 permanent magnet with 0.126 A\cdotm\(^{-2}\) dipole magnitude and Allegro A1392 Hall-effect sensors) in a lubricated rigid tube (inner diameter 19.1 mm). Three commercial water-based lubricants of different viscosities were used to simulate variations in the environment that may be encountered in the intestines: a liquid (Equate™ Personal Lubricant Liquid), a thin jelly (Equate™ Personal Lubricant Jelly), and a thick jelly (Kroger® Lubricating Jelly). The experiments consisted of two parts (Fig. 2). To estimate \( \alpha \) and \( \beta \), we applied constant force (via gravity) and no torque to the capsule by dropping the capsule through the tube at a 31° incline. The resulting average linear and angular velocity were measured via video recording at 60 FPS. To measure angular velocity, the capsule was marked with lines indicating fixed angular increments. Both average velocities were computed as the total distance traveled divided by the total time. 10 trials were conducted for each lubricant and an average was taken over these trials, and the measured velocity-force correspondences were used to estimate \( \alpha \) and \( \beta \). To

![Fig. 1 An applied force \( f \) and torque \( \tau \) on a screw-type capsule results in a linear velocity \( v \) and angular velocity \( \omega \). All four vectors are assumed to be aligned with the lumen.](image-url)
estimate \( \gamma \), we used tri-axial Helmholtz coils (described in [9]) to apply a rotating uniform magnetic field to the capsule with constant angular velocity 0.4 Hz and constant field strength (5 mT) sufficient to maintain the capsule in synchronous rotation. Under these conditions, a constant torque and no force is applied to the capsule and torque measurements can be used with the observed angular velocity to estimate \( \gamma \). Alternatively, we use a relationship that does not require torque measurements (which is valid only under zero-force conditions)

\[
\gamma = \frac{\alpha \omega}{\beta} 
\]  

(2)

to compute \( \gamma \) from an observation of both the linear and angular velocity of the capsule. Using this relationship, it is possible to determine \( P^{-1} \) even for capsules that do not have embedded magnetic sensors. 10 trials were conducted for each lubricant and average velocities were taken over these trials. As the capsule advanced through the tube, the tube was moved in the opposite direction to keep the capsule in the central workspace of the Helmholtz coils.

**RESULTS**

The estimated inverse propulsion matrices for the environments simulated by each of the three qualitatively distinct lubricants are given below.

\[
P_{\text{liquid}}^{-1} = \begin{bmatrix} 11.9 & 231 \\ 231 & 197927 \end{bmatrix}
\]

\[= 197927 \begin{bmatrix} 6.0 \times 10^{-5} & 0.0012 \\ 0.0012 & 1 \end{bmatrix}\]  

(3)

\[
P_{\text{thin jelly}}^{-1} = \begin{bmatrix} 8.82 & 153 \\ 153 & 109114 \end{bmatrix}
\]

\[= 109114 \begin{bmatrix} 8.1 \times 10^{-5} & 0.0014 \\ 0.0014 & 1 \end{bmatrix}\]  

(4)

\[
P_{\text{thick jelly}}^{-1} = \begin{bmatrix} 5.65 & 106 \\ 106 & 79666 \end{bmatrix}
\]

\[= 79666 \begin{bmatrix} 7.1 \times 10^{-5} & 0.0013 \\ 0.0013 & 1 \end{bmatrix}\]  

(5)

The matrices above are also shown with the largest parameter (i.e., \( \gamma \)) factored out in the form

\[
P^{-1} = \gamma \begin{bmatrix} \alpha / \gamma & \beta / \gamma \\ \beta / \gamma & 1 \end{bmatrix} 
\]  

(6)

As we move from the least viscous “liquid” to the most viscous “thick jelly”, we observe a dramatic increase in each of \( \alpha, \beta, \) and \( \gamma \), with each reducing by a factor of 2, approximately. However, in the factored form, we observe a relatively small change in \( \alpha / \gamma \) and \( \beta / \gamma \), with no clear increasing or decreasing trend. We further observe that, if we simply assume constant average values of \( \alpha / \gamma = 7.1 \times 10^{-5} \) and \( \beta / \gamma = 0.0013 \) across lubricant types, we would find that the error between each of the experimentally determined parameters and their respective average is at most 17%.

**DISCUSSION**

Our results suggest that it is reasonable to express the propulsion matrix of a given screw-type capsule in a given environment, which has three independent parameters, as the product of a constant matrix and a single scalar parameter that varies as a function of the environment. The scalar parameter appears to be proportional to the environmental fluid viscosity, as expected based on the analogous model for microswimmers. The two-parameter constant portion, as well as a good initial guess for the factored term, can be determined experimentally (offline)—using a single environment, or results can be averaged over a small number of environments—and the scalar parameter can then be fitted to new environments previously unencountered. The scalar parameter could be allowed to evolve in time to update the propulsion matrix as the capsule encounters variations in properties throughout the lumen, which will likely lead to improved performance of model-based controllers.

**REFERENCES**


